KU LEUVEN

Inhibiting the natural number bias in rational number tasks: Towards a comprehensive test instrument

Jo Van Hoof, Rianne Janssen, Lieven Verschaffel, & Wim Van Dooren

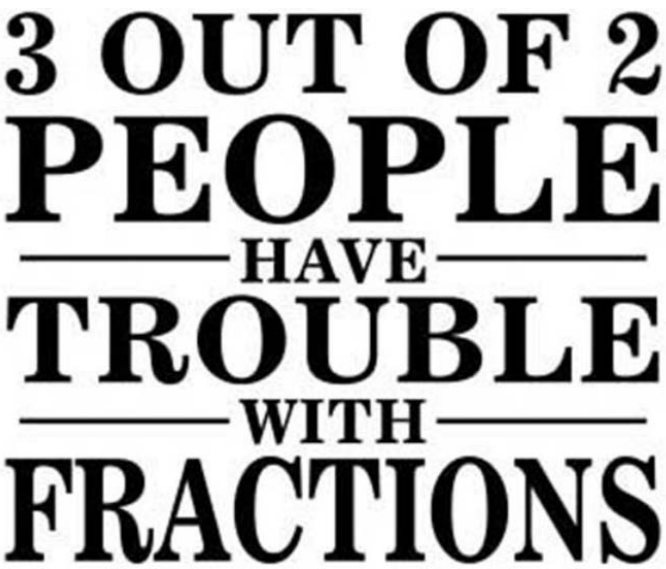
Centre for Instructional Psychology and Technology, University of Leuven

jo.vanhoof@ppw.kuleuven.be

Theoretical background

Understanding rational numbers = essential part of numerical development

Students have a lot of difficulties with rational numbers (fractions and decimal numbers)



KU LEUVEN

Theoretical background

- **The Natural Number Bias** (NNB)

  = inappropriately applying natural number properties in tasks with rational numbers

- Three main aspects on which natural and rational numbers differ
  - Density
  - Size
  - Operations

KU LEUVEN

Density

- Unlike natural numbers, rational numbers do not obey the successor principle (Vosniadou & Vamvakoussi, 2006)

- Example:

  → How many numbers are there between 0,2 and 0,5?

  answer: 2?

Size

- “longer decimals are larger” and “shorter decimals are smaller” (Resnick et al., 1989).

- Wrong assumption that a fraction’s numerical value increases when its denominator, numerator, or both increase (Mamede, Nunes, & Bryant, 2005)

- Example

  → 0,25 > 0,7

  → $\frac{7}{12} > \frac{3}{4}$

Operations

- The effect of operations on rational numbers is different from that on natural numbers (Prediger, 2006)

- Example

→ addition or multiplication makes bigger

$50*0,7 > 50$

→ subtraction or division makes smaller

$25 : 0,4 < 25$

KU LEUVEN

Development of a comprehensive test instrument

Problem statement

- No comprehensive test that includes all 3 aforementioned aspects

→ *How are the three different aspects related to each other?*

KU LEUVEN

The development of a comprehensive test instrument

- Extensive literature review and analysis of Flanders' school curriculum → creation of a comprehensive paper-and-pencil test

- Two types of items

Congruent: use of natural number knowledge leads to correct answer

Incongruent: use of natural number knowledge leads to incorrect answer

- Test characteristics

  - Positive and negative numbers
  - Different representations of rational numbers
  - Varying degrees of difficulty
  - Open and multiple choice questions

KU LEUVEN

Examples of the items

- <u>Density</u>

  Write a number between 3,2 and 3,7

  Write a number between 1,2 and 1,3

  How many numbers are there between 3,4 and 3,5?

- Size

Rank-tasks

Put these numbers from small to large and circle the ones that have the same size

0,5 - 0,21 - 0,784 - 0,3 - 0,97 - 0,125 – 0,30 - 0,85 – 0,365 – 0,72

Kleinste:	.....	.....	.....	.....	.....	.....	.....	.....	Grootste: .....

Comparison-tasks

Choose the largest number:

$\frac{3}{7}$ or $\frac{11}{15}$

$\frac{2}{6}$ or $\frac{5}{17}$

KU LEUVEN

Operations

- Is “50* $\frac{3}{2}$” smaller of larger than 50
- Is “26,3 * 0,4” smaller or larger than 26,3
- What is half of $\frac{1}{8}$?

Data collection

4th graders

- 9 schools, 213 pupils
- No time limit (25-40 min)
- 53 items (34 incongruent and 19 congruent)

Total test

- 21 schools (9 in primary education, 12 in secondary education) from the different parts of Flanders
- A total number of 1343 students (4th, 6th, 8th, 10th and 12th grade)
- 179 items (137 incongruent, 42 congruent)

KU LEUVEN

Research questions

1. Natural number bias?

2. Is this bias present to the same extent for all three aspects?

3. Is the ability to overcome the natural number bias a multidimensional/unidimensional construct? (1 latent variable?)

↳ Only incongruent items

1 + 2 → Generalized Estimation of Equations (GEE, logistic regression)

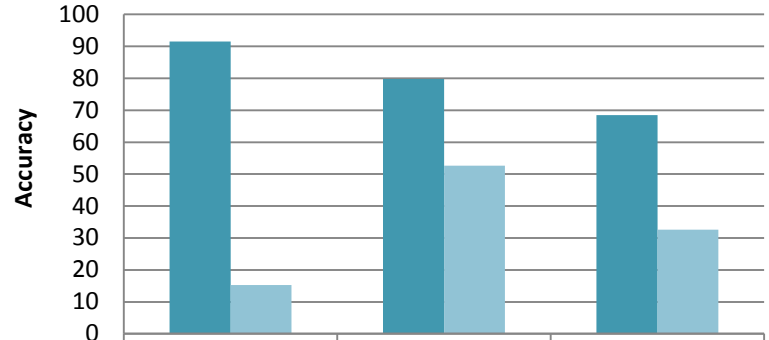
3 → Item Response Theory (IRT)

Analysis – GEE (logistic regression)

- Cronbach's alpha = 0,86

- Significant **main effect of congruency**, $X^2(1, N=11289)=531{,}647$, $p<.001$
  - Congruent: 78,7%
  - Incongruent: 41,7% → ***NNB!***
  - OR = 5,148

- Significant **interaction effect between congruency and aspect**, $X^2(3, N=11289)=96{,}681$, $p<.001$
  - OR density = 59,804
  - OR size = 3,558
  - OR operations = 4,496



	density	size	operations
congruent	91,5	79,8	68,5
incongruent	15,3	52,6	32,6

→ ***NNB = different per aspect***

KU LEUVEN

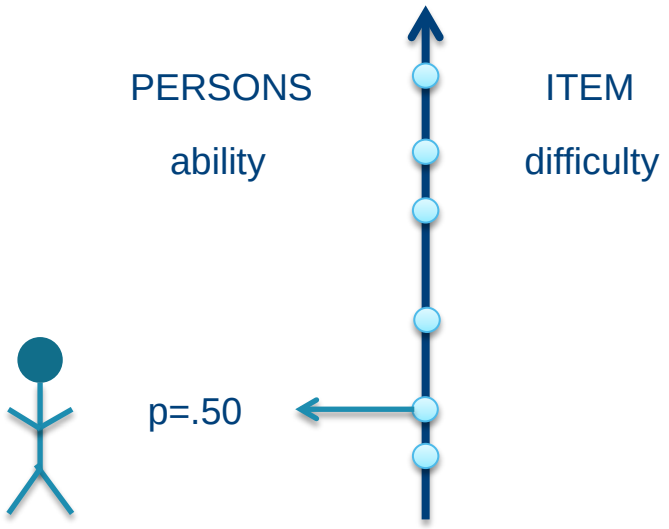
Analysis – Item Response Theory (IRT)

Rasch model

→Most simple IRT model

→Probability of a correct response= function of the difference between

1. Person ability parameter
2. Item difficulty parameter



$$P(X_{ij} = 1) = f(\theta_j - \beta_i)$$

Analysis – Rasch model

- **Unidimensionality assumption**

  Development = continuous growth along one common path
  → the underlying latent variable

- **Local independence assumption**

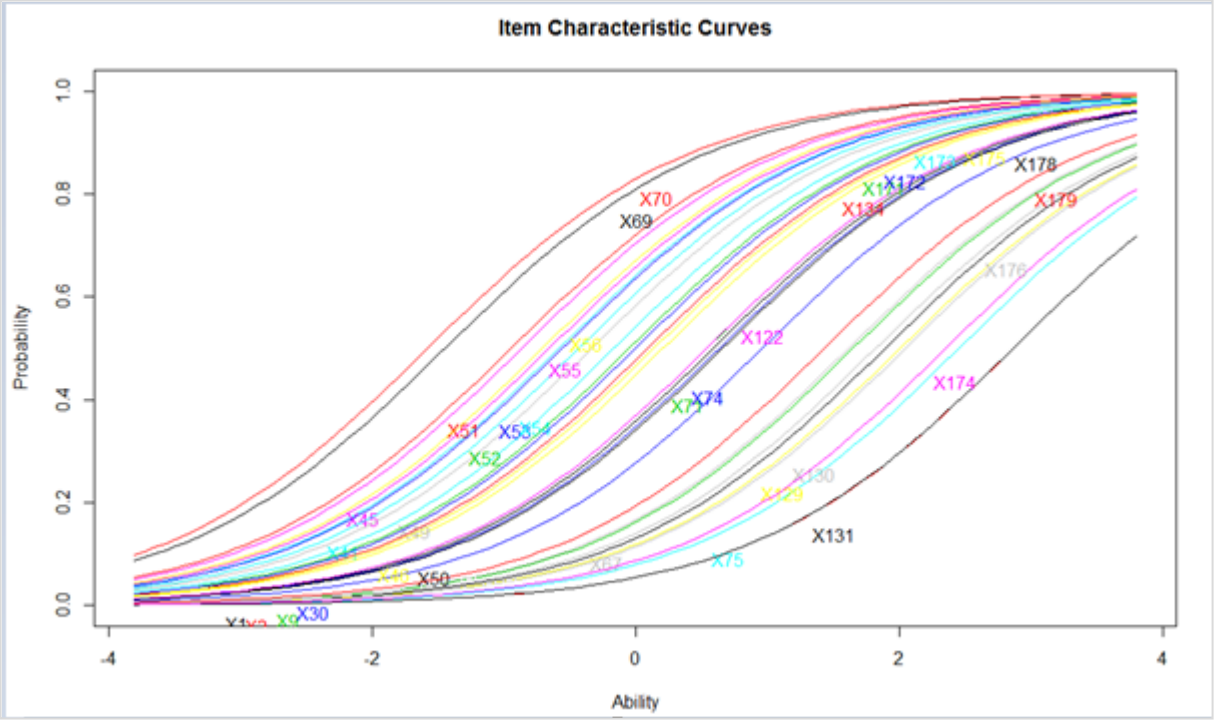
  The chance of answering an item correctly is independent of the outcome of any other item after controlling for person and item parameters

Results – Rasch model

- Bootstrap Goodness-of-Fit using Pearson chi-squared

  # data-sets = 200

  p-value = 0,57 → *good model fit!*



KU LEUVEN

Density

Size

Operations

Is 21 : 0,7 bigger or smaller than 21?

0,36 – 0,2 = …

Conclusions & discussion

- Overall NNB in 4th graders, strongest in density items
- Inhibiting the natural number bias is a one-dimensional construct in 4th graders
- Density items are more on top of the scale, size items are more at the bottom (but a few on top!)

→ Differences at item level between and within aspect!

KU LEUVEN

Conclusion and discussion

- We created a valid instrument that measures how good 4th graders are in inhibiting their natural number knowledge in tasks with rational numbers

- <u>Advantages</u>
  - It is possible to estimate a person's ability level by the person's response pattern to a subset of items

    → Useful in further research on "natural number inhibition" (e.g. international comparison, longitudinal research)

  - When the item difficulty is known, it is possible to calculate the probability of success on this item for every value of ability (pupil) (Embretson & Reise, 2000)

  - It is possible to relate 'natural number inhibition' with other aspects of number sense, or other student characteristics

KU LEUVEN

Thank you for your attention!